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DIFFUSION APPROXIMATIONS FOR THE COOPERATIVE SERVICE OF VOICE A--ETC(U)

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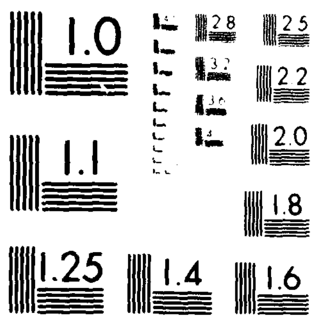
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NPS55-80-007

NAVAL POSTGRADUATE SCHOOL
Monterey, California

DIFFUSION APPROXIMATIONS FOR THE COOPERATIVE SERVICE OF VOICE AND DATA MESSAGES

by

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and

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February 1980

Approved for Public Release; Distribution Unlimited.

Prepared for:

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14 NPS55-80-007	2. GOVT ACCESSION NO. AD-A085126	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 Diffusion Approximations for the Cooperative Service of Voice and Data Messages,		5. TYPE OF REPORT & PERIOD COVERED 9 Technical rept.,
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 J. P. Lehoczky [redacted] D. P. Gaver		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		12. REPORT DATE 11 Feb[redacted] [redacted]80
		13. NUMBER OF PAGES ~~30~~ 12 42
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Queues
Communications
Data Transmission
Voice Transmission
Semigroup Theory
Probability Modeling

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A probability model is presented for a set of communication channels that share the service of data and voice transmissions. A diffusion-theoretic approximation is derived, utilizing new results of Burman (1979). It is shown that the data queue (which is of low priority relative to voice) is approximated by a Wiener process.

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S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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INTRODUCTION

In this paper we study the behavior of a queueing system which arises in the study of certain communication networks. Specifically we study a queueing phenomenon which arises with the SENET network, as described by Coviello and Vena (1975) or Barbacci and Oakley (1976). This network allows for both voice and data messages to be transmitted over the same channels by using a special type of integrated circuit and packet-switched multiplexor structure. The two classes of traffic have substantially different performance requirements. Voice messages tend to possess great redundancy, and hence not to be sensitive to channel error rates, while data is very sensitive to channel error, having essentially no redundancy. Voice messages on the other hand have critical timing requirements and cannot be queued, while data is relatively insensitive to timing and can be queued. Additionally, voice messages tend to be very long relative to data messages which can be broken up into small packets. These special requirements have led to the following queueing network. A node of the network consists of $c + v$ channels or servers. The voice messages are assigned to v channels and do not queue. Thus the voice messages operate as a loss system. Data messages may use c channels exclusively and any unused voice channels; however, voice preempts data using voice channels. Data messages are queued if necessary. Typical performance measures that one may wish to calculate include the loss rate of voice traffic and the mean data queue length.

We make standard probabilistic assumptions. Specifically, we assume voice traffic arrives according to a Poisson(λ) process and each voice message has an independent exponential(μ) service time. Data messages are assumed to have independent exponential(η) service times and arrive according to a Poisson(δ) process. With these assumptions voice is an M/M/v/v loss system, and data is an M/M/S system where $S = c + v - V(t)$ with $V(t)$ = number of voice messages in service. The stochastic process $\{(X(t),V(t)),\ t \geq 0\}$ is Markov with state space $Z^+ \times \{0,1,\ldots,v\}$ where $X(t)$ = data system size at time t. One can easily write the Kolmogorov forward equations appropriate for this system; however, these equations do not yield a closed form solution. To describe this system one must either numerically solve the forward equations or introduce approximations.

This system has been studied previously by a number of researchers including Halfin and Segal (1972), Halfin (1972), Fischer and Harris (1976), Bhat and Fischer (1976), Fischer (1977), Chang (1977), and Gaver and Lehoczky (1979a,b). The last two papers introduce a "fluid flow" and a diffusion approximation and derive explicit formulas for data queue behavior. These papers focus on the important case in which $\rho_d = \delta/\eta > c$. In such a situation the data messages must have access to voice channels for the system to be stable. Furthermore, it was assumed that η/μ was large, say 10^4. Under these circumstances the data flow could be treated deterministically. Suppose we define $\rho_v = \lambda/\mu$ and $q = (\rho_v^v/v!)/\sum_{j=0}^{v} \rho_v^j/j!$, the Erlang B blocking probability. The total traffic intensity on the $c + v$ channels is given by $\rho_d + \rho_v(1-q)$, or we could define $\rho = (\rho_d + \rho_v(1-q))/(c+v)$. A heavy traffic approximation can be derived for this case $\rho \nearrow 1$. Such an approximation was derived in Gaver and Lehoczky (1979b) assuming η/μ was large; a Wiener process with reflecting boundary was found appropriate. In this paper we derive a heavy traffic approximation for the system without the fluid flow assumption that η/μ is large. The methodology is drawn heavily from the approach of Burman (1979). In this approach one characterizes a Markov process by its infinitesimal generator. One next suitably normalizes the process so that the generator converges to a limiting infinitesimal generator (in this case to that of a reflected Brownian motion). This convergence allows the conclusion that the finite dimensional distributions of the normalized Markov process converge. The diffusion approximation consists of treating the actual process through its limiting behavior. The details are somewhat complicated by the presence of a boundary.

2.

Let $\{(X(t),V(t)),\ t \geq 0\}$ be a bivariate Markov process with state space $S = Z^+ \times \{0,1,\ldots,v\}$. Here $\{V(t),\ t \geq 0\}$ is marginally an M/M/v/v loss system with arrival rate λ and service rate μ. Conditional on $V(t)$, $\{X(t),\ t \geq 0\}$ is an $M/M/(c + v - V(t))$ queueing system with arrival rate δ and service rate η. We say that the V process subordinates the X process. We let

$$Q = \mu \begin{pmatrix} -\rho_v & \rho_v & & & & \\ 1 & -(1+\rho_v) & \rho_v & & & \\ & 2 & -(2+\rho_v) & \rho_v & & \\ & & \ddots & \ddots & \ddots & \\ & & & & & \rho_v \\ & & & v-1 & -(v-1+\rho_v) & \rho_v \\ & & & & v & -v \end{pmatrix}, \quad (2.1)$$

the infinitesimal generator of the V process.

The generator of the (X,V) process is given by

$$Af(x,k) = \begin{cases} Qf(x,k) + \delta(f(x+1,k) - f(x,k)) \\ \qquad + \eta(c+v-k)(f(x-1,k)-f(x,k)) \\ \qquad\qquad \text{if} \quad x \geq c+v-k \\ \\ Qf(x,k) + \delta(f(x+1,k) - f(x,k)) \\ \qquad + \eta x(f(x-1,k) - f(x,k)) \\ \qquad\qquad \text{if} \quad x < c+v-k \end{cases} \quad (2.2)$$

for $f: S \to R$ continuous where

$$Qf(x,k) = \rho_v f(x,k+1) - (k+\rho_v)\, f(x,k) + kf(x,k-1) \qquad v \geq k \geq 0 \tag{2.3}$$

and $f(x,-1) = f(x,v+1) = 0$. Clearly $Qf(x) = 0$, that is Q annihilates functions of x alone. We next normalize the (X,V) process by defining $X_n(t) = X(nt)/\sqrt{n}$ and $V_n(t) = V(nt)$. One can calculate the generator of the Markov process $\{(X_n(t), V_n(t)),\ t \geq 0\}$ having state space $S_n = \{0, 1/\sqrt{n}, 2/\sqrt{n}, \ldots\} \times \{0,1,\ldots,v\}$ to be

$$A_n f(x,k) = \begin{cases} nQf(x,k) + \delta n(f(x + 1/\sqrt{n},k) - f(x,k)) \\ \qquad + \eta_n (f(x - 1/\sqrt{n},k) - f(x,k)) \\ \qquad\qquad \text{if } x \geq \dfrac{c + v - k}{\sqrt{n}} \\ nQf(x,k) + \delta n(f(x + 1/\sqrt{n},k) - f(x,k)) \\ \qquad + \eta n \sqrt{nx}\ (f(x + 1/\sqrt{n},k) - f(x,k)) \\ \qquad\qquad \text{if } x = 0, 1/\sqrt{n}, \ldots, (c+v-k)/\sqrt{n}\ . \end{cases} \tag{2.4}$$

We assume $f(x,k)$ has three bounded derivatives in x for each fixed k. With this assumption one can expand terms in (2.4) in a Taylor series and rewrite as

$$A_n f(x,k) = \begin{cases} nQf(x,k) + n^{1/2} f_x(x,k)(\delta - \eta(c+v-k)) \\ \qquad + \frac{1}{2} f_{xx}(x,k)(\delta + \eta(c+v-k)) \\ \qquad + O(n^{-1/2}) \\ \qquad \text{if } x \geq (c+v-k)/\sqrt{n} \\ nQf(x,k) + n^{1/2} f_x(x,k)(\delta - \eta\sqrt{nx}) \\ \qquad + \frac{1}{2} f_{xx}(x,k)(\delta + \eta\sqrt{nx}) + O(n^{-1/2}) \\ \qquad \text{if } x = 0, 1/\sqrt{n}, \ldots, (c+v-k)/\sqrt{n} \end{cases} \tag{2.5}$$

with $f_x(x,k) = \frac{\partial}{\partial x} f(x,k)$ and $f_{xx}(x,k) = \frac{\partial^2}{\partial x^2} f(x,k)$.

We ultimately wish to prove that the finite dimensional distributions of $\{X_n(t), t \geq 0\}$ converge to those of a Wiener process with reflecting barrier at the origin. This can be restated in terms of semi-groups. We let $\{T_t^n, t \geq 0\}$ be the semi-group of operators associated with $\{(X_n(t), V_n(t)), t \geq 0\}$ and $\{T_t^\infty, t \geq 0\}$ be that associated with a Wiener process having reflecting barrier at 0. Let g be a continuous function $g: R' \to R'$. Knowledge of the semi-group is equivalent to knowledge of the transition functions by taking a sequence of g's which approximate indicator functions. We wish to prove $|T_t^n g(x,k) - T_t^\infty g(x)| \to 0$ as $n \to \infty$ for all (x,k). Here $T_t^n g(x,k) = E(g(X_t) | X_N(0) = x, V_n(0) = k)$. The presence of the variable k prevents this from being done directly. The method we use is to construct a convenient sequence of functions $\langle g_n \rangle_{n=1}^{\infty}$ which converge in some sense to g. We write

$$\|T_t^n g - T_t^\infty g\|_n \le \|(T_t^\infty g)_n - T_t^\infty g\|_n + \|T_t^n g - T_t^n g_n\|_n + \|T_t^n g_n - (T_t^\infty g)_n\|_n \qquad (2.6)$$

where $\| \ \|_n$ refers to the sup norm over S_n. Both T_t^n and T_t^∞ are contraction semigroups.

$\langle (T_t^\infty g)_n \rangle_{n=1}^{\infty}$ is the sequence of functions constructed from $T_t^\infty g$. Our goal is to show that each of the three terms on the right side of (2.6) converges to 0. The first and second terms can be handled similarly. For any function g, we must guarantee that the constructed $\langle g_n \rangle_{n=1}^{\infty}$ sequence satisfies $\|g_n - g\|_n \to 0$. It will follow that $\|(T_t^\infty g)_n - T_t^\infty g\|_n \to 0$. Moreover, since $\{T_t^n, t \ge 0\}$ is a contraction semi-group $\|T_t^n g - T_t^n g_n\|_n \le \|g - g_n\|_n$ which also converges to 0. The sequence $\langle g_n \rangle_{n=1}^{\infty}$ will be chosen in such a way that the third term converges to 0.

We focus on a convergence determining class of functions g, those which are bounded and have three bounded derivatives. For such a function $g(x)$ we define

$$g_n(x,k) = g(x) + \frac{1}{\sqrt{n}} u(x,k) + \frac{1}{n} v(x,k) \qquad (2.7)$$

where u and v have two bounded derivatives in x for each fixed k. The functions u and v will be determined explicitly later and are chosen to control the third term in (2.6). Clearly when g_n is defined by (2.7), $\|g_n - g\|_n = O(n^{-1/2})$ and therefore converges to 0 as required.

One can apply the generator A_n to g_n to derive

$$A_n g_n(x,k) = \begin{cases} nQg(x) + n^{1/2}[Qu(x,k) + g'(x)(\delta - \eta(c+v-k))] \\ \quad + [Qv(x,k) + u_x(x,k)(\delta - \eta(c+v-k)) \\ \qquad + \frac{1}{2} g''(x)(\delta - \eta(c+v-k))] + O(n^{-1/2}) \\ \qquad\qquad \text{if } x \geq (c+v-k)/\sqrt{n} \\ \\ nQg(x) + n^{1/2}[Qu(x,k) + g'(x)(\delta - \eta n^{1/2}x)) \\ \quad + [Qv(x,k) + u_k(x,k)(\delta - \eta n^{1/2}x) + \frac{1}{2} g''(x)(\delta + \eta n^{1/2}x)] \\ \quad + O(n^{-1/2}) \\ \qquad\qquad \text{if } x = 0, 1/\sqrt{n}, \ldots, (c+v-k)/\sqrt{n} \end{cases}$$

where $u_x(x,k) = \frac{\partial}{\partial x} u(x,k)$. Recall that Q annihilates functions of x alone, thus $nQg(x) \equiv 0$. We want to have $A_n g_n(x,k)$ converge to a finite limit and to have that limit be independent of k. For this to occur, the $n^{1/2}$ term must be controlled and the functions u and v must be chosen in such a way as to eliminate the variable k.

The $n^{1/2}$ coefficient in (2.8) can be rewritten by adding and subtracting

$$\sum_{k=0}^{v} \pi_k g'(x) \quad (\delta - \eta(c+v-k)) = -\eta(c+v)(1-\rho) g'(x) \quad .$$

We next pick $u(x,k)$ to be a solution of

$$\begin{aligned} \Omega u(x,k) &= -(g'(x) \; \eta(\rho_d - (c+v-k)) + g'(x) \; \eta(c+v)(1-\rho)) \\ &= -g'(x) \; \eta(k-\rho_v(1-q)) \qquad (2.9) \end{aligned}$$

When $u(x,k)$ is any solution of (2.9), the coefficient of the $n^{1/2}$ term in (2.8) becomes

$$\begin{aligned} &-g'(x) \; \eta(c+v)(1-\rho) && \text{if} \quad x \geq \frac{c+v-k}{\sqrt{n}} \\ &g'(x) \; \eta((c+v)\rho - n^{1/2}x - k) && \text{if} \quad 0 \leq x < \frac{c+v-k}{\sqrt{n}} \end{aligned}$$

Equation (2.9) can be solved explicitly. Define $a_k = -g'(x) \; \eta(k-\rho_v(1-q))/\mu$, so that (2.9) can be written as

$$\begin{aligned} -\rho_v(u(x,k)-u(x,k-1))-(k-1)(u(x,k-1)-u(x,k-2)) &= a_{k-1}, \; k = 1,\ldots,v \\ -v(u(x,v)-u(x,v-1)) &= a_v \qquad (2.10) \end{aligned}$$

Equation (2.10) has a solution since $\sum_{k=0}^{v} \pi_k a_k = 0$, where $\langle \pi_k \rangle_{k=0}^{v}$ is the stationary distribution associated with Q, or $\pi_k = (\rho_v^k/k!)/(\sum_{i=0}^{v} \rho_v^i/i!)$. The solution is given by

$$u(x,k) - u(x,k-1) = \frac{\sum_{i=0}^{k-1} \pi_i a_i}{\rho_v \pi_{k-1}} = \frac{-g'(x)\eta T_{k-1}}{\mu \rho_v \pi_{k-1}}$$

where

$$T_k = \sum_{i=0}^{k} \pi_i (1-\rho_v(1-q)) \quad \text{and} \quad T_v = 0.$$

Clearly

$$u(x,k) = u(x,0) - \frac{g'(x)\eta}{\mu\rho_v} \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}}, \quad 1 \le k \le v \qquad (2.11)$$

where $u(x,0)$ is arbitrary. We let $u(x,0) = \frac{1}{2} g'(x)$ so

$$u(x,k) = g'(x)\left(\frac{1}{2} - \frac{\eta}{\mu\rho_v} \sum_{i=1}^{k} \frac{T_{i-1}}{\pi_{i-1}}\right), \quad 0 \le k \le v \qquad (2.12)$$

For the choice of u specified by (2.12) we next wish to insure that the limiting generator is independent of the variable k. The function v is chosen to eliminate the dependence on k. The $O(1)$ term of (2.8) is given, for $x \ge (c+v-k)/\sqrt{n}$, by

$$Qv(x,k) + g''(x)\left[\frac{1}{2} - \frac{\eta}{\mu\rho_v}\sum_{i=1}^{k}\frac{T_{i-1}}{\pi_{i-1}}(\delta - \eta(c+v-k))\right]$$

$$+ \frac{1}{2}g''(x)(\delta + \eta(c+v-k))$$

$$= Qv(x,k) + H(x,k).$$

Let $\bar{H}(x) = \sum_{k=0}^{v} \pi_k H(x,k)$ and consider $Qv(x,k) + (H(x,k) - \bar{H}(x)) + \bar{H}(x)$. We now let $v(x,k)$ be any solution of

$$Qv(x,k) = -(H(x,k) - \bar{H}(x)) \quad . \tag{2.13}$$

Equation (2.13) has a one-parameter family of solutions, since $\sum_{k=0}^{v} \pi_k(H(x,k) - \bar{H}(x)) = 0$. When $v(x,k)$ is chosen to be any solution of (2.13), the $O(1)$ term of (2.3), for $x \geq (c+v-k)/\sqrt{n}$, will become $\bar{H}(x)$ and will therefore be independent of k. It remains to calculate $\bar{H}(x)$.

$$\bar{H}(x) = g''(x)\left[\sum_{k=0}^{v}\pi_k\left\{\left(\frac{1}{2} - \frac{\eta}{\mu\rho_v}\sum_{i=1}^{k}\frac{T_{i-1}}{\pi_{i-1}}\right)(\delta - \eta(c+v-k))\right.\right.$$

$$\left.\left.+ \frac{1}{2}(\delta + \eta(c+v-k))\right\}\right]$$

$$= g''(x)\left[\delta - \frac{\eta}{\mu\rho_v}\sum_{k=0}^{v}\pi_k(\delta - \eta(c+v-k))\sum_{i=1}^{k}\frac{T_{i-1}}{\pi_{i-1}}\right] \tag{2.14}$$

$$= g''(x)\left[\delta - \frac{\eta^2}{\mu\rho_v}\sum_{k=0}^{v}\pi_k(k - \rho_v(1-q))\sum_{i=1}^{k}\frac{T_{i-1}}{\pi_{i-1}}\right.$$

$$\left.+ \frac{\eta(c+v)(1-\rho)}{\mu\rho_v}\sum_{k=0}^{v}\pi_k\sum_{i=1}^{k}\frac{T_{i-1}}{\pi_{i-1}}\right].$$

The second term can be rewritten by interchanging the order of summation. The third term is $O(1-\rho)$. We find

$$\bar{H}(x) = g''(x)\left[\delta - \frac{\eta^2}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_i}{\pi_i}\sum_{k=i+1}^{v}\pi_k(k-\rho_v(1-q)) + O(1-\rho)\right]$$

$$= g''(x)\left[\delta - \frac{\eta^2}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_i}{\pi_i}(T_v-T_i) + O(1-\rho)\right]$$

with $T_v = 0$ or

$$\bar{H}(x) = g''(x)\,\eta\left[\rho_d + \frac{\eta}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_i^2}{\pi_i} + O(1-\rho)\right] . \qquad (2.15)$$

For the functions u and v specified by (2.12) and (2.13), equation (2.8) can be rewritten as

$$A_n g_n(x,k) = \begin{cases} -n^{1/2}(1-\rho)(\sigma+v)\,\eta g'(x) \\ \quad + \eta\left[\rho_d + \frac{1}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_i^2}{\pi_i} + O(1-\rho)\right] g''(x) + O(n^{-1/2}) \\ \qquad\qquad \text{for } x \geq (c+v-k)/\sqrt{n} \\ \\ n^{1/2}\,\eta[(c+v)\rho - n^{1/2}x - k]g'(x) \\ \quad + \eta g''(x)\left[\rho_d + \frac{\eta}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_i^2}{\pi_i} + O(1-\rho)\right. \\ \qquad \left. - (c+v-k-n^{1/2}x)\frac{\eta}{\mu\rho_v}\sum_{i=0}^{k}\frac{T_i}{\pi_i}\right] + O(n^{-1/2}) \\ \qquad\qquad \text{for } x \leq (c+v-k)/\sqrt{n} \end{cases} \qquad (2.16)$$

We now introduce the "heavy traffic approximation." In order for the generator to converge to a limiting generator we must have $1-\rho = O(n^{-1/2})$. Specifically, we assume $\rho = \rho_n = 1 - (\theta/\sqrt{n})$ for some $\theta \geq 0$. In this case, $n^{1/2}(1-\rho) = \theta$, and (2.16) becomes

$$A_n g_n(x,k) = \begin{cases} -\theta\eta(c+v)g'(x) + \eta\left[\rho_d + \dfrac{\eta}{\mu\rho_v}\displaystyle\sum_{i=0}^{v-1}\dfrac{T_i^2}{\pi_i}\right]g''(x) + O(n^{-1/2}) \\ \qquad \text{for} \quad x \geq (c+v-k)/\sqrt{n} \\ \eta[(c+v)\rho - n^{1/2}x - k]n^{1/2}g'(x) \\ \quad + \eta g''(x)\left[\left\{\rho_d + \dfrac{\eta}{\mu\rho_v}\displaystyle\sum_{i=0}^{v-1}\dfrac{T_i^2}{\pi_i}\right\} \right. \\ \quad \left. - (c+v-k-n^{1/2}x)\dfrac{\eta}{\mu\rho_v}\displaystyle\sum_{i=0}^{k}\dfrac{T_i}{\pi_i}\right] \\ \qquad \text{for} \quad x \leq (c+v-k)/\sqrt{n} \end{cases} \tag{2.17}$$

We now define a limiting generator A_∞ with domain consisting of all functions g having three bounded derivatives and $g'(0) = 0$. Let

$$A_\infty g(x) = -\theta\eta(c+v)\ g'(x) + \eta\left[\rho_d + \frac{\eta}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_i^2}{\pi_i}\right]g''(x), \qquad x > 0 \tag{2.18}$$

A_∞ is the generator of a Markov process which corresponds to a Wiener process with drift $-\theta\eta(c+v)$, scale

$$2\eta\left[\rho_d + \frac{\eta}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_v^2}{\pi_i}\right],$$

and a reflecting barrier at 0. The $O(n^{-1/2})$ terms involve the first three derivatives of g which are bounded. It is clear from a direct comparison of (2.16) and (2.18) that $|A_n g_n(x,k) - A_\infty g| \to 0$ as $n \to \infty$ for all $x > 0$ and k arbitrary. In addition, $g'(0) = 0$ is necessary for the generator to converge at $x = 0$. Unfortunately even assuming $g'(0) = 0$,

$$|A_n g(0,k) - A_\infty g(0)| \to (c+v-k)\frac{\eta^2}{\mu\rho_v}\sum_{i=0}^{k}\frac{T_i^2}{\pi_i}g''(0) \quad \text{as} \quad n \to \infty$$

rather than to 0. One needs a special argument to handle this lack of convergence at the boundary.

We set out to prove the third term in (2.6) converges to 0. Standard semi-group results (see Burman, 1979, p. 33) give

$$(T_t^\infty g)_n - T_t^n g_n = \int_0^t T_{t-s}^n((A_\infty w)_n - A_n w_n)\,ds \tag{2.19}$$

where $w = w(t,x) = T_t^\infty g(x)$. Recall that $w_n = w + (1/\sqrt{n})u + (1/n)v$ with u and v defined by (2.12) and (2.13) with g replaced by w. It follows that

$$\|T_t^n g_n - (T_t^\infty g)_n\|_n$$

$$= \left\|\int_0^t T_{t-S}^n((A_\infty w)_n - A_\infty w + A_\infty w - A_n w_n)\,dS\right\|_n$$

$$\leq \int_0^t \|T_{t-S}^n((A_\infty w)_n - A_\infty w)\|_n dS + \left\|\int_0^t T_{t-S}^n(A_\infty w - A_n w_n)\,dS\right\|_n$$

$$\leq \int_0^t \|(A_\infty w)_n - A_\infty w\|_n dS + \left\|\int_0^t T_{t-S}^n(A_\infty w - A_n w_n)\,dS\right\|_n .$$

The first term is clearly $O(n^{-1/2})$. It remains to show that the second is $O(n^{-1/2})$ as well. We have shown $|A_\infty w - A_n w_n| = O(n^{-1/2})$ except at the boundary where it is $O(1)$. We split the integral into two parts, for one of which the process is away from the boundary, and for the other, near the boundary. The integral away from the boundary has an integrand which is $O(n^{-1/2})$. The integral near the boundary is also $O(n^{-1/2})$ since under a heavy traffic assumption the process is rarely near the boundary. The details are merely summarized here; they are based on the ideas of Burman (1979).

Let I_{on} be the indicator function of

$$\left[0, \frac{c+v-k}{\sqrt{n}}\right)$$

and I_{1n} be the indicator of

$$\left[\frac{c+v-k}{\sqrt{n}}, \infty\right) \quad .$$

We have

$$\left\| \int_0^t T^n_{t-S}(A_\infty w - A_n w_n)\,dS \right\|_n$$

$$\leq \left\| \int_0^t T^n_{t-S}(A_\infty w - A_n w_n) I_{1n}\,dS \right\|_n + \left\| \int_0^t T^n_{t-S}(A_\infty w - A_n w_n) I_{0n}\,dS \right\|_n$$

$$\leq \left\| \int_0^t (A_\infty w - A_n w_n) I_{1n}\,dS \right\|_n + \| A_\infty w - A_n w_n \|_n \left\| \int_0^t T^n_{t-S} I_{0n}\,dS \right\|_n \quad .$$

The first term is $O(n^{-1/2})$, since $|A_\infty w - A_n w_n| = O(n^{-1/2})$ off the boundary. The factor $\| A_\infty w - A_n w_n \| = O(1)$, thus it remains to show that

$$\left\| \int_0^t T^n_{t-S} I_{0n}\,dS \right\|_n = O(n^{-1/2}) \quad .$$

This gives the total time in $[0,t]$ spent near the boundary.

We bound

$$\left\| \int_0^t T^n_{t-S} I_{0n}\,dS \right\|_n$$

by first introducing a function $h(x)$ not in the domain of A_∞. We let $h(x)$ have bounded support, be infinitely differentiable and be given by $h(x) = x$ for x near 0. One can construct $h_n(x)$ using (2.7) and apply A_n to h_n to find

$$A_n h_n = \begin{cases} O(1) & \text{if } x \geq \dfrac{c+v-k}{\sqrt{n}} \\ n^{1/2}\eta((c+v)\rho - n^{1/2}x-k) + O(1) & \text{if } x < \dfrac{c+v-k}{\sqrt{n}} \end{cases} \tag{2.20}$$

One has

$$T_t^n h_n - h_n = \int_0^t T_S^n A_n h_n dS$$

$$= \int_0^t T_S^n A_n h_n I_{1n} dS + \int_0^t T_S^n A_n h_n I_{0n} dS .$$

It follows that

$$\left\| \int_0^t T_S^n A_n h_n I_{0n} dS \right\|_n \leq \| T_t^n h_n - h_n \|_n + \left\| \int_0^t T_S^n A_n h_n I_{1n} dS \right\|_n$$

$$\leq 2\|h_n\|_n + O(1) \ .$$

We have shown $\left\| \int_0^t T_S^n A_n h_n I_{0n} dA \right\|_n$ to be bounded in n. An application of (2.2) shows

$$\left\| \int_0^t T_S^n A_n h_n I_{0n} dA \right\|_n = n^{1/2}\eta\,|(C+v)\rho - n^{1/2}x - k + O(1)|\,\left\| \int_0^t T_S I_{0n} ds \right\|_n$$

is bounded in n. It follows that $\left\| \int_0^t T_S^n I_{0n} dS \right\|_n = O(n^{-1/2})$.

This finally concludes the argument which shows $\|T_t^n g_n - (T_t^\infty g)_n\|_n = O(n^{-1/2})$, hence by (2.6) $\|T_t^n g - T_t^\infty g\|_n = O(n^{-1/2})$.

We have thus shown that the finite-dimensional distributions of the $(X_n(t), V_n(t))$ process converge to those of a Wiener process with drift $-\theta\eta(c+v)$ scale

$$\eta\left(\rho_d + \frac{\eta}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_i^2}{\pi_i}\right),$$

and reflection at 0. The diffusion approximation treats $X_n(t)$ as though it were such a Wiener process. For instance, the limiting Wiener process has a stationary exponential distribution with parameter

$$\frac{\theta(c+v)}{\rho_d + \frac{\eta}{\mu\rho_v}\sum_{i=0}^{v-1}(T_i^2/\pi_i)}.$$

This is a distribution for $X(nt)/\sqrt{n}$ and suggests $X(t)$ will have a steady state distribution given approximately by an exponential with parameter

$$(c+v)(1-\rho)\bigg/\left(\rho_d + \frac{\eta}{\mu\rho_v}\sum_{i=0}^{v-1}\frac{T_i^2}{\pi_i}\right).$$

The steady state mean data queue length would then be

$$E(X(t)) = \frac{\rho_d + \frac{\eta}{\mu\rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i}}{(c + v)(1-\rho)} . \tag{2.21}$$

It is interesting to consider the special case $c = 0$, $v = 1$ where the two types of traffic use the same channel. Under heavy traffic $\rho = \rho_d + \rho_v/(1+\rho_v)$, so $\rho_d \approx (1+\rho_v)^{-1}$. The mean data queue length derived from the diffusion approximation (2.21) will be

$$\left(\rho_d + \frac{\eta}{\mu} \frac{\rho_v}{(1+\rho_v)^3}\right) \Big/ (1-\rho) \approx \frac{\rho_d}{1-\rho}\left(1 + \frac{\eta}{\mu} \frac{\rho_v}{(1+\rho_v)^2}\right) .$$

The latter is the exact expression derived by Fisher (1978) for this case. The expression (2.21) represents a generalization of the results of Gaver and Lehoczky (1979b). In this paper, a diffusion approximation is given based on the fluid flow assumption for the data. For this case the result is the same except that the scale is given by

$$\frac{\eta^2}{\mu\rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i}$$

rather than

$$\eta\left(\rho_d + \frac{\eta}{\mu\rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i}\right).$$

The results derived in this paper therefore definitely generalize the results of Gaver and Lehoczky (1979b) since the variability in the data queue is now included. When η/μ is large, the second term dominates, and the fluid flow approximation is satisfactory.

The Wiener process approximation for the $X(t)$ process provides a method for studying the dynamics of that process. For instance, suppose the data queue were at level x at time t where x is large. One might wish to study the time that elapses until the queue becomes empty. This is essentially the duration of the busy period under heavy traffic and corresponds to a first-passage time for a Wiener process. Let us denote it by T_x. Straightforward martingale arguments provide for its transform

$$E(e^{-sT_x}) = \exp\left[\left(\frac{x}{\sigma}\right) - \left(\frac{m}{\sigma}\right) - \sqrt{\left(\frac{m}{\sigma}\right)^2 + 2s}\right] \tag{2.22}$$

where

$$m = \theta(c+v)\eta \approx n^{1/2}(1-\rho)(c+v)\eta$$

$$\frac{\sigma^2}{2} = \eta\left(\rho_d + \frac{\eta}{\mu\rho_v}\sum_{i=0}^{n-1}\frac{T_i^2}{\pi_i}\right).$$

It is also easy to find the mean first-passage time

$$E(T_x) = x/m \tag{2.23}$$

One might also be interested in the area beneath the sample path until emptiness occurs, since this area represents the total time waited by all data customers involved in the busy period. If A_x represents this area, simple backward equation arguments give

$$E(A_x) = \frac{x^2}{2m} + \frac{\sigma^2}{2m^2} x \tag{2.24}$$

where m and σ^2 are given in (2.22).

Acknowledgment. This research was supported in part by a contract from the Office of Naval Research.

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